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# Modelling Compliant Mechanisms – Comparison of Models in MATLAB / SimMechanics vs. FEM

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**Abstract**. The paper deals with modelling flexible structures of compliant mechanisms in MATLAB / SimMechanics. The analysis is based on building most adequate models of flexible hinges that replace revolute joints of mechanisms. Main point of the paper is to provide a powerful tool for analysis and initial design of elastic mechanisms based on compact structures. Using proposed mathematical models enables to evaluate kinematic and dynamic properties of particular joints and the mechanism, as well. Simulation of flexural behaviour for most general joints and the gripper mechanisms are calculated in MATLAB / SimMechanic and results are compared with calculations using FEM method. Both calculations are made for the same forms, dimensions and input parameters.

**Keywords**. Compliant mechanisms, flexure hinge (joint), modelling and simulation, FEM,

## 1. Introduction

General trends in the development of technical components and devices are characterized by requirements on higher efficiency and decreasing size and energy consumption. In cases of manipulation devices with objects smaller than 1mm a high positioning accuracy is required. Mechanisms for these purposes are no more constructed from discrete components (joints, arms, etc.), but the whole mechanisms is created as compact elastic structure made from a piece of material (Howell, 2001; Smith, 2000).

Compliant mechanisms can be divided in two groups: structures with concentrated and distributed flexibility. In the first group of mechanisms the rigid parts / arms are connected by flexible joints. The second group, beside rigid parts, includes flexible joints and flexible arms too. In this paper the mechanisms with elastic joints only will be discussed. Main focus will be oriented on the of plane mechanisms problems solution. The main objective is to propose some possible approaches and solutions can be applied in the initial design of such flexible structures. Presented approach is based on using universal SW tools for technical calculations MATLAB and integrates analyses (kinematic and dynamic) into one simulation model. The results of simulations represent input parameters for design of control system. Similar mechatronic approach to design of compliant mechanisms for robotic devices is discussed in (Howell, 2001; VDI2206, 2004; Chen and Lin, 2004; We-Shing, 2003).

## 2. Model of flexible joint

Solving flexible mechanism as the first step it is necessary to build the basic block: the model of the flexural joint. MATLAB and its toolbox SimMechanics is a suitable tool for modelling flexural structures. This program is directly designed for solution of kinematics and dynamics of the connected mechanical bodies. Another advantage is that the program designer does not require knowledge of motion equations of the mechanism. For creation of the model only basic design information such as topology, dimensions of relevant elements and basic inertia characteristics expressed by weight and matrices for moments and inertia are sufficient.

The disadvantage of the chosen simulation environment is the lack of options for calculation of problems focused on elasticity and rigidity of the body. The compact structures belong to this problem group too, as the representation by mathematical description of mass-spring-damper system is used. Another problem of the toolbox is the absence of the parasite joint deflections, but the error in the calculation has no fundamental meaning for mechanisms that consist of rigid elements. On the other hand in cases of elastically compliant joints the parasite / cross deflections have major influence on the total accuracy of the positioning mechanisms. Partial solution of this problem is outlined.

Building models of flexible joint in the MATLAB / SimMechanic and its modifications were previously described in (Chudnovsky et al, 2006; Hricko, Harťanský and Havlik, 2009). The principle is based on splitting the joint into small parallel independent elements and the assumption that the joint deflections are within limit of linear elastic deformation i.e. for area of the Hooke's law.

The principal block diagram of the joint model represents Figure 1. Solution of a given problem lies in the segmentation of the beam (joint) on $n$ elastic segments. Each of these segments consists of elementary three components: the rigid coupling C - connector, the deformable elements B - body and the core CE - core of the elasticity. Mutual connection of two rigid components C (in one segment) is made through the stiffness matrix. So, for the planar case two lateral elastic deflections (as sliding motions) in directions $x$ and $y$ and one rotation around the $z$ axis should be calculated. One side of the segment – body B is strongly fixed to the imaginary frame and the force, calculated from the total load, affects on the second side of the segment. The transformations of farces and displacements are made through the calculation block *Weld*, which is imaginarily connected to the end of model. This segmentation enables to calculate the direct interaction between the solid parts of the mechanism and flexible segments / joints.

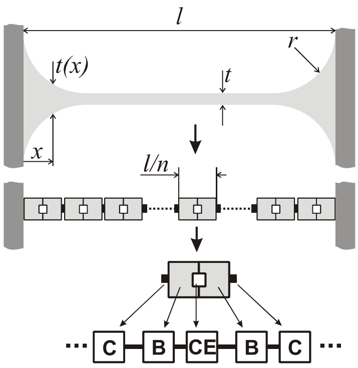


Fig. 1. Basic block diagram of flexible joint model

The flexural behaviour of one element / flexible joint describes dynamic equation

$$Mu'' + Bu' + Ku = F(t) \qquad (1)$$

where:

- $M$, $B$, $K$ are matrices mass, damping and stiffness properties respectively
- $u$ is the deflection vector that consists of two lateral and angular displacements.
- *F(t)* is the load - time variable vector.

The input parameters of the flexible joint model are: Young's elasticity modulus $E$, material density $\rho$, joint width $w$, thickness of the notch - the smallest thickness of the joint $t$, parameters describing the shape of the notch (as for instance: circular notch - radius $r$, for coroner filleted hinge - curvature radius $r$ and length of the joint $l$; if $l=2r$, then we get from coroner-filleted joint right circular, and parameter $r$ is this same for both joints and for parabolic joint $c$ and $l$ parameters) (Lobontiu, 2003).

Components of matrices $M$, $B$, $K$ should be calculated in the initialization phase the calculation process.

### 2.1. Compliance / stiffness matrix

There are numerous publications dealing with calculations of the stiffness / compliance matrices for flexural joints (Lobontiu, 2003; Schotborgh et al, 2005; Yong, Lu and Handley, 2008). For solving planar problem elements of the flexibility matrix (compliance) are evident from equation (2) that represents the relations between acting load and resulting deformation (Lobontiu, 2003)

$$\begin{pmatrix} u_x \\ u_y \\ \theta_z \end{pmatrix} = \begin{pmatrix} C_{xFx} & 0 & 0 \\ 0 & C_{yFy} & C_{yMz} \\ 0 & C_{\theta zFy} & C_{\theta zMz} \end{pmatrix} \begin{pmatrix} F_x \\ F_y \\ M_z \end{pmatrix} \qquad (2)$$

### 2.2. Mass matrix

Properties for inertial moment of are not specified in SimMechanic environment when the blocks from the Joint group are used. Since our model consists of several elements that compose Body blocks and Custom Joint, it is necessary to specify the mass and inertia matrix in the block Body. The meaning of particular components in the inertia matrix is known. For the mass the one element the following relation can be used

$$m_i = \rho w \int_{\frac{l}{n}(i-1)}^{\frac{l}{n}i} t(x)dx; \quad i = 1,2,\ldots n \qquad (3)$$

where: $n$ is the number of elements, *t(x)* is the function describing the shape of the notch. For symmetrical form of a joint one get the equivalent function. An example of the symmetrical joint and equivalent function waveform *t(x)* is shown in Figure 2.

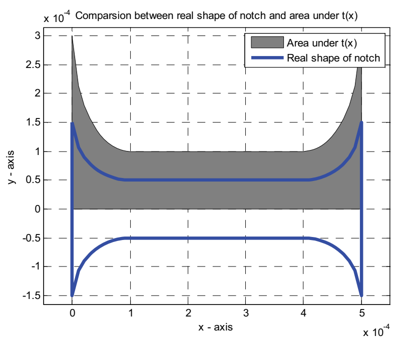


Fig. 2. Symmetric flexible joint (line) and area under function *t(x)*

To determine the mass matrix for the flexible joint well known methods are usually used. Therefore, this mass matrix will have the following form

$$M = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_z \end{pmatrix} \tag{4}$$

where $m$ is the total weight of the flexible joint and $I_z$ is the moment of inertia around the $z$ axis (Smith, 2000; Musil, 2006).

**2.3. Damping matrix**

Calculation of the damping matrix is the relatively complex tasks since the exact approach is based on measurements damping parameters / characteristics for a given material. Because of these characteristics are usually not known they are replaced by estimated values of so called proportional damping. The most common substitution for the damping matrix is known as Rayleigh's damping defined as a linear combination of mass and stiffness matrices (Musil, 2006)

$$B = \alpha M + \left( \beta + \gamma \frac{sign(\omega)}{\omega} \right) K \tag{5}$$

where: $\alpha$ is the external viscous damping coefficient, $\beta$ is the coefficient of Voigt's model of the internal damping and $\gamma$ represents Sorokin frequency independent model of internal damping. In other literature (He and Fu, 2001, Chowdhury and Dasgupta, 2003) the relation (5) is presented without damping coefficient $\gamma$. According to (Chowdhury and Dasgupta, 2003), the coefficients $\alpha$ and $\beta$ are related as follows

$$\zeta_i = \frac{\alpha}{2\omega_n} + \frac{\beta\omega_n}{2} \tag{6}$$

where $\zeta_i$ is the factor of modal damping factor and $\omega_n$ is the natural frequency. The disadvantage of this relation (6) is that two variables are unknown. A suitable solution seems to be in rewriting the equation (6) for two selected frequencies in order to obtain the system of equations. Then, both constants $\alpha$ and $\beta$ can be calculated from this system of equations. However, these constants will correspond to selected frequencies only.

According to (Adhikari, 2006) it is possible to determine damping matrix as the product of inversions of the mass and the stiffness matrices: $f\left(M^{-1}K, K^{-1}M\right)$ or $f\left(KM^{-1}, MK^{-1}\right)$. Some other definitions of the damping matrix are described in above mentioned literature. For expression of damping matrix the first two members of the decomposition function can be used too. According to above discussion for calculation of the damping matrix that respects the proportional damping two following relations can be used

$$B = Me^{-\left(M^{-1}K\right)^2/2} \sinh\left( K^{-1}M \ln\left(M^{-1}K\right)^{2/3} \right) + K\cos^2\left(K^{-1}M\right)\sqrt[4]{K^{-1}M}\,\tan^{-1}\frac{\sqrt{M^{-1}K}}{\pi} \tag{7}$$

$$B = M\left( \frac{a_0}{2} I + \sum_{r=1}^{\infty} \left[ a_r \cos\left( \frac{2\pi r}{\Omega} \sqrt{M^{-1}K} \right) + b_r \sin\left( \frac{2\pi r}{\Omega} \sqrt{M^{-1}K} \right) \right] \right) \tag{8}$$

**2.4. Compensations in model**

As mentioned before, disadvantage of the SimMechanics toolbox is not only the absence of tools for solution of elasticity and stiffness problems. The toolboxes do not provide possibilities for solution of mathematical model for real elastic bodies / joints with the parasite strains and deformations. The effect of parasite deformation on the overall behaviour of a joint is mathematically expressed by the flexibility/stiffness matrix with of-diagonal elements. Unfortunately, the impact of these cross - stiffness coefficients result in non-negligible deflections of the joint. As for the case when the joint is forced by the moment around the $z$ axis. If the solution is calculated using program SimMechanics the effect of rotation around this axis is obtained only. The influence of $C_{yMz}$ coefficient that results in displacement in y-axis is omitted. So, results of such simulation are not exact.

For this reason the solution to this problem lies in insertion of the compensation elements by two ways:

- Compensation of loads directly acting on the joint
- Adjustment of offsets for loads

The first way is based on the assumption of validity of Hooke's law. Considering (2), the compensation should include multiplication of the force $F_y$ by the two matrix elements of compliance matrix and the result is added to the acting torque $M_z$. Similarly, the value of the applied moment $M_z$ is changed. The block scheme for this compensation is shown in Figure 3.

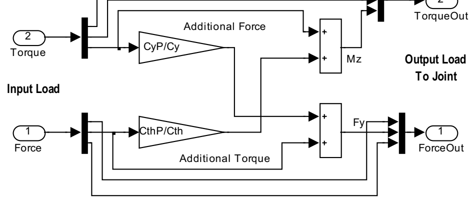


Fig. 3. Block scheme of compensation for direct load to joint.

The second type of compensation is more complex solution and cannot be solved as elegant as in the previous case. In this case, the resulting force is calculated using transformation of applied force and torque from the point where the load acts. Insertion of the Joint Sensor blocks enables to get compensated values for the forces and moments. Then the force $F_y$ and moment $M_z$ are multiplied by already known constants. This procedure can be formulated as follows: Obtain the force / moment on the flexible joint independently on the actual joint position (reaction force / torque is the function of displacement / rotation of the joint). Ensure that the additional force / torque do not act on other parts of the mechanism. This requirement contains the contradictory task. The joint should be isolated from the whole mechanism.

Partial solution of the first task lies in total compensation by Weld joints where force/moment reactions are obtained. Main drawback is that these data are equivalent to the rigid mechanism. The only problem is in partial isolation for various joints and subsequent solution of kinematics and dynamics for the whole structure.

## 3. The kinematic structure

The example of a kinematic structure is shown in Figure 4 a). This is the small end-effector based compact elastic structure. Basic input data for design and model are: material - aluminium, with a stress modulus of 70 GPa, density 2700 kg/m3, Poisson's ratio 0.35. Parameters of all right-circular hinges (joints) are radius of notch 0.5 mm, notch thickness 0.05 mm. Parameters of corner-filleted joints are: corner-filleted radius 0.1 mm, length 0.97 mm, thickness of the joint 0.1 mm. Thickness of whole mechanisms is 0.5 mm, external dimensions of whole mechanisms are 9.71x5 mm (height x width). The gripper is designed for grasping objects with dimensions about 0.2 mm.

Modelling and simulation of this gripper mechanism was made in MATLAB / SimMechanics environment and 2D drawing is shown in Figure 4 b). At the beginning no exact shape definition of the mechanism was given. The model was created only by linking the basic building blocks such as bodies (red colour) and flexible joints (blue colour). The ground blocks are only in block scheme, and are not shown on the final visualization.

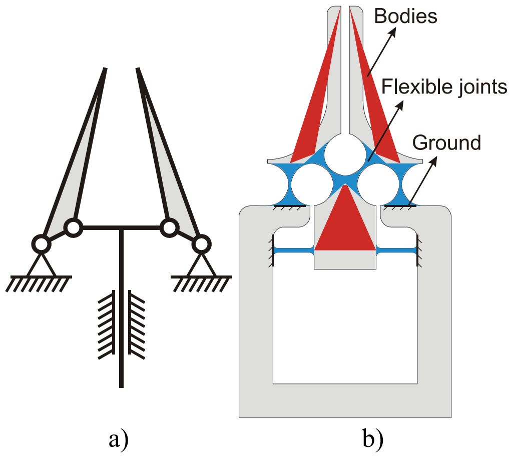


Fig. 4. a) Kinematic scheme of end-effecter, b) Comparison of models visualization 2D draft (grey) and SimMechanics (blue and red parts)

### 3.1. FEM models

Program based on principle of the finite element method (FEM) COMSOL Multiphysics was chosen for verification of results in MATLAB / SimMechanics. FEM software was used as reference tool and as a substitution of real measurements. In this environment some models were built. Models are solved in two study types: Stationary and Time Dependent. For the stationary study were get final deflections of mechanisms. Time Dependent and dynamics of device was studied as time step response. Results from the stationary study were used for verification in time dependent behaviour, i.e. when the object will be in stable state (it is defined: time of study, start time, step and end time). To get equal results with MATLAB models the linear solver was used in both studies.

In order to verify results obtained for the single joint loaded by the force and torque. Observed deformations are indicated in figure 5 a). The point of acting force / torque and place of fixed constraint is shown in figure 5 b). Positions of points *A* and *B* show the real (simulated) displacements. From positions of A and B points the angle of bending (rotation) $\theta_z$ was calculated

$$\theta_z = \arctan\frac{A(y)-B(y)}{A(x)-B(x)} \tag{9}$$

If the distances between points A and B is too small It could be said that results are not, in principle, determined.

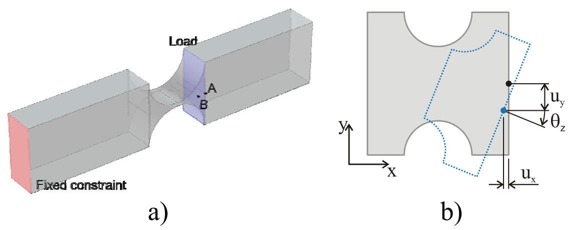


Fig. 5. a) Specification of joint load and fixed constraint b) Specification (formulation) of analysed deflection

The model of gripper mechanisms with the place of actuator shows Figure 6. For visualisation of displacements and comparison of kinematic behaviour the positional data of the point C are used calculated.

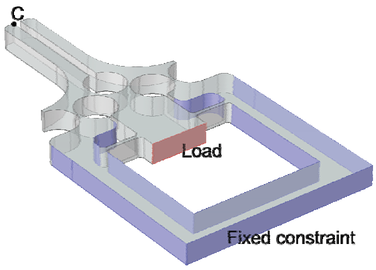


Fig. 6. The gripper mechanisms with the place of actuator

### 3.2. Results and comparisons

At the beginning of the verification process the results obtained only from single joint where compared. The loading force was applied on free end of the joint (force / torque). Obtained results are summarized in Table 1, where the percentage error between the linear solver in the FEM and the solver in MATLAB / SimMechanics are compared according to the formula

$$Err_{\%} = \frac{|u_{SM}| - |u_{FEM}|}{0.01|u_{FEM}|} \quad (9)$$

where $u_{SM}$ represents obtained deformation from the model calculated in SimMechanics and $u_{FEM}$ represents deformation obtained from the model calculated in FEM environment. Compensation for directly applied load to the flexible joint is already considered in presented results.

Tab. 1. Results comparison of flexible joint between MATLAB model and FEM model

| Load | Deformation | $Err_{\%}$ [%] |
|---|---|---|
| $F_x$ | $u_x$ | -3.95 |
| $F_y$ | $u_y$ | 4.76 |
| $M_z$ | $\theta_z$ | 30.54 |
| $F_x$, $F_y$ | $u_x$ | 3.98 |
| | $u_y$ | 4.78 |
| $F_y$, $M_z$ | $u_y$ | 3.22 |
| | $\theta_z$ | -2.64 |

As can be seen from these data the error due to the action of a single moment about the $z$ axis is much greater than expected. This value can be verified by chosen method for calculation of angular displacement only $\theta_z$. This rotation in FEM was calculated from displacement of two near points and on the free end of the joint. Similar results in comparison of analytical solutions were reached by other authors too (Yong, Lu and Handley, 2008; Lobontiu, 2002).

Both simulations of the kinematic structure in MATLAB and FEM are shown in Figure 7. Visualization in MATLAB does not enable to see the opening of fingers due to emergence of small distortions. But the displacements of fingers is evident from deflections of joints in x and y axes where the gaps between segments of the joint can be seen.

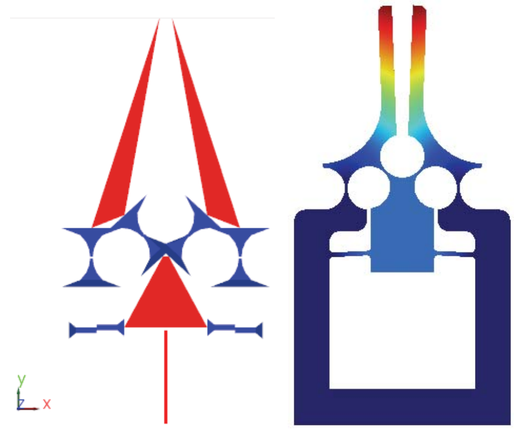


Fig. 7. Models visualization in state of open fingers

Figure 8 shows the gripper dynamics and the movement of the reference point as the response to a step input.

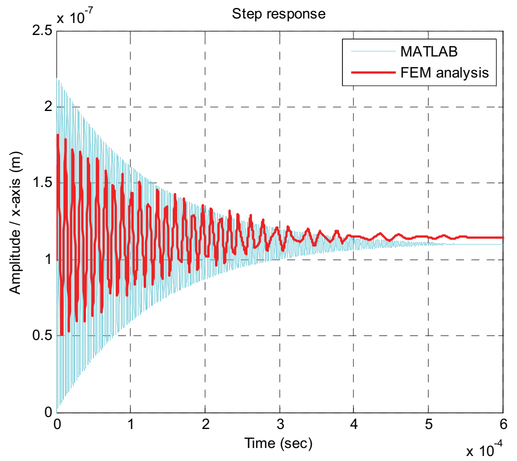


Fig. 8. Step response of the model from MATLAB (higher frequency) and FEM software

From comparison of both trajectories one can conclude that there is a final - static error between the results from MATLAB and FEM analysis. It is obvious that the transition processes are different which may be a consequence of the method was chosen for estimation of the damping matrix. Rem.: the damping matrix based on theory of proportional damping can be applied for some selected dynamic modes only.

## 4. Conclusion

This paper is focused on study of behaviour of elastically compliant mechanisms using SimMechanics environment. Mathematical model of flexible joint is presented. Using this model it is possible to design the geometry and parameters for desired flexural characteristics of an elastic joint.

It is briefly presented the new concept of SW tool specified for study and initial design of compact elastic structures. It enables modelling elastic mechanisms and simulations of main kinematic and dynamic characteristics of joints and the whole structure. The main motivation and purpose of this work was to use the MATLAB toolbox SimMechanics for analysis and design of compliant mechanisms for robotic devices. Application of the proposed approach is shown on design of the small gripper mechanism made from the elastic body. Results from models built in MATLAB / SimMechanics environment are verified using FEM analysis. Comparisons of results are discussed. As follows results and discussion there are two tasks for future research: compensation of parasite deflections and the study of dynamics including suitable damping matrix which largely affects the dynamic behavior of the whole mechanism

## 6. Acknowledgments

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## 7. References

Chudnovsky, V.,et al., (2006) Modeling Flexible Bodies in SimMechanics and Simulink. <http://www.mathworks.com/company/newsletters/digest/2006/may/simmechanics.html>

Hricko, J., Harťanský, R., Havlík, Š., (2009) Modeling Compliant Mechanical Joints for Micro-Robotic Devices, In: *RAAD 2009: 18th International Workshop on Robotics in Alpe-Adria-Danube Region*, ISSN 2066-4745, Bucuresti: Printech

Lobontiu, N. (2003) *Compliant Mechanisms: Design of Flexure Hinges*. Florida: CRC Press LLC, 447p. ISBN 0849313678.

Schotborgh O. W. et al, (2005) Dimensionless design graphs for flexure elements and a comparison between three flexure elements, In: *Precision Engineering*, Vol. **29**, pp. 41–47

Yong, K. y., Lu, T-F., Handley, D.C. (2008) Review of circular flexure hinge design equations and derivation of empirical formulations, *Precision Engineering* Vol. **32**, ISSN:0141-6359, 63–70

Musil, M. (2000) *Parameters identification of oscillate mechanical systems* / Identifikácia parametrov kmitajúcich mechanických sústav, Habilitation work, STU Bratislava, 2000 [In Slovak]

He, J., Fu, Z-F., (2001), *Modal Analysis*, Elsevier Ltd., ISBN: 978-0-7506-5079-3

Chowdhury I, Dasgupta S.P., (2003), Computation of Rayleigh damping coefficients for large systems. *The Electronic Journal of Geotechnical Engineering*, Vol. **8C**, ISSN 1089-3032

Adhikari, S. (2006), Damping modelling using generalized proportional damping, *Journal of Sound and Vibration* Vol. **293**, ISSN: 0022-460X, 156–170

Lobontiu, N. et. al. (2002) Parabolic and hyperbolic flexure hinges: flexibility, motion precision and stress characterization based on compliance closed-form equations, In: *Precision Engineering*, Vol. **26**, pp. 183–192

Howell, L. L., (2001), *Compliant Mechanisms*. [s.l.] : Wiley-IEEE, 459 s. ISBN 047138478X.

Smith, T.S., (2000), *Flexures: elements of elastic mechanisms*.: Gordon and Breach Science Publishers, 448 s. ISBN 90-5699-261-9

VDI2206. Entwicklungsmethodik für mechatronische Systeme. Düsseldorf : VDI-Verlag, 2004. 110s.

Chen, W. J. and LIN, W.,(2004): Design of a Flexure-Based Gripper Used in Optical Fiber Handling In: *2004 IEEE International Conference on Robotics, Automation and Mechatronics (ICRAM 2004)*, Singapore, 1-3 December 2004, pp. 83-88.

We-Shing, Hung (2003). *A Study on the Design and Analysis of microgripper for Microassembly* [online]. [s.l.], 2003. 131 s. National Sun Yat-Sen University . Dostupné z WWW: <http://etd.lib.nsysu.edu.tw/ETD-db/ETD-search/view_etd?URN=etd-0710103-144403>.