Modeling Robotic (Micro) Mechanisms With Flexural Parts

Havlík Štefan
Institute of Informatics, Slovak Academy of Sciences,
Severná 5, 974 01 Banská Bystrica, Slovakia
E-mail: havlik@savbb.sk

Abstract

The paper deals with a class of robots where classical connections of kinematic elements are replaced by flexural joints / arms. By this way it is possible to design small compact kinematic structures and to achieve a high positioning accuracy of the system, as whole. These usually parallel structures exhibit some special features as regards to mechanical performance and control possibilities. To achieve desired features of such a robotic – general spring like mechanisms the careful design process and mechatronic approach should be applied. This approach includes techniques flexural analysis for complex structures, methods of parameter optimization and synthesis, evaluation of the system performance as well as possible control.

1. Introduction

New applications of robotic systems in microelectronics, optoelectronics, medicine, biology, etc. require to develop new task oriented robotic structures that exhibit specific performance together with development of sophisticated manufacturing technologies, sensing and control methods. There are several specific requirements that follow from particular application tasks. For instance:

- Extremely high positioning accuracy, which is usually better then 1 μm. It is obvious that this accuracy can be achieved within a limited range of motions only.
- Small or limited volume of the operation space, frequently some few cm^3, or usually less then 1 mm^3 in cases of micro-mechanisms.
- Contact force / grasp force reflecting or control capabilities.

As obvious, classic constructions of robot mechanics based on assembly from discrete mechanical parts can not satisfy desired features as to accuracy, volume of operation space as well as high force reflecting sensitivity. There are several examples of tasks and solutions where desired performances are achieved by compact design and constructions of such task oriented mechanisms [1,2,4]. This was enabled by the development of MEMS technologies and other high precision manufacturing methods [1]. Naturally more complex mechanical structures that include elastic elements suppose using careful techniques for design, kinematic, force and compliance analyses. Despite the different scale factors, comparing micro and "normal scale" mechanisms, there are common methods for performing analyses and calculations. On the other hand, although mechanisms with flexural joints exhibit no friction and backlash effects, functional deflections of particular elastic parts in mechanisms play much more important role then in conventional mechanisms. It should be said that the elastic – flexural connection is no "ideal" kinematic pair as represent revolute or prismatic joints. Any flexural joint, besides the desired axis of motion which corresponds to d.o.f., exhibits some cross flexural effects what deteriorates accuracy of the mechanisms, as whole. An approach to solving complex elastic structures, analysis and modeling are briefly presented.

2. Parts of Flexible Mechanisms

Any complex mechanical structure can be considered as composition of mutually interconnected body segments. The mechanical segment is then described by its geometry, mass, its flexural characteristics and force / displacement interaction with respect to other neighboring segments. It should be said the difference from FEM (Finite Elements Method) approach that consists in level of segmentation, i.e. each segment will correspond to its form as a part of mechanisms directly specified by geometrical dimensions. By other words: FEM method can be applied for flexural analysis of particular segments, or, as well as for the whole structure too. Comparing to robotic structures segments in our consideration are particular arms, links, joints, …etc.

Mechanical segments

There are, in principle, segments that exhibit some elastic compliance and relatively rigid segments. Compliant segments are usually made as thin profiled straight or curved sections, beams that crate elastic joints or arms. Some typical and experimental forms and their kinematic representations are shown in Fig. 1. [1,2,3]

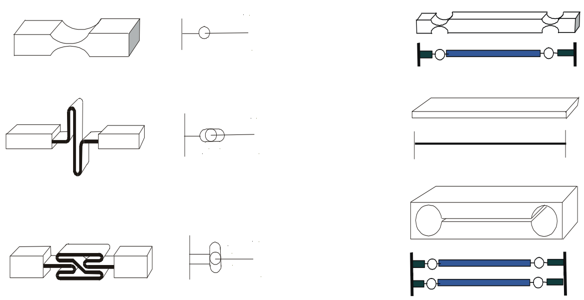


Fig. 1. Some forms of flexible joints and arms

Mechanisms

The mechanisms consist of flexible joints or arms and rigid parts. As regards to kinematic structure ranges of desired motions and degrees of freedom should correspond to flexural compliance limits of particular joints or arms. As obvious, these limits result in relatively small operation workspace comparing to classic kinematics with revolute or prismatic joints. Examples of some kinematic structures are in Fig.2.

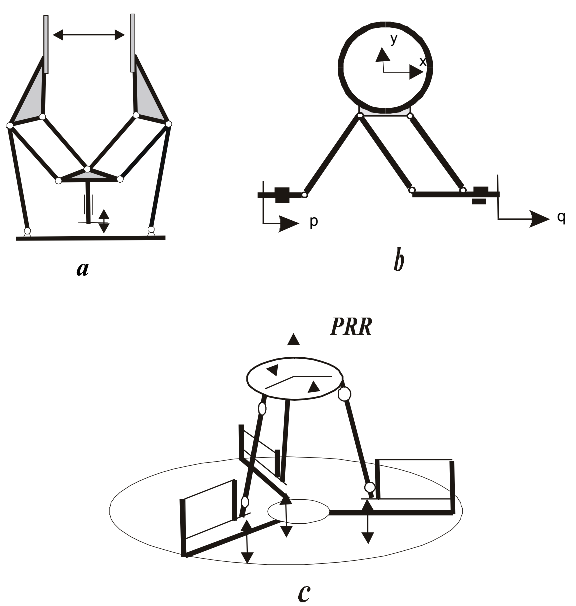


Fig. 2. Some examples of mechanisms with flexible joints a) gripper [7], b) x-y positioning table, c) PRR positioning table [1]

Problems to study

Comparing to classic robot structures the kinematics based on flexible mechanisms exhibit several important features have to be studied. There are:

– Any end position / configuration of mechanism which corresponds to a set of input "joint" variables is related to a level of internal energy of elasticity. Then, the working space can be mapped by potential functions and displacements as results of positive / negative work between them. Actual stiffness / compliance of the mechanisms should be evaluated.
– Actuating forces are given by superposition of variable forces need for flexural displacement and available forces as result of external loads. The force analysis and statement the workspace limits are required.
– Kinematic transformations that relate displacements of actuated parts (control variables) and end coordinates depend on stiffness/compliance characteristics. Methods of verification these functions for direct and inverse tasks should be found and applied. These methods will be naturally related to the sensing principles used for positional measurements.
– The final task is building the complex model of the flexure that will enable to optimize parameters and form of flexural segments and its whole geometry during the design phase as well as simulation and control.

3. Analysis of Kinematic Flexures

By the same way as in rigid robotic linkages we distinguish main reference spaces:

- The joint space where vectors of actuated variables i.e. displacements and driving forces / torques are defined.
– The end of "arm" reference system usually related to a tool or a gripper.
– The global reference system.

The task is to perform flexural / kinematic analysis of a complex structure i.e. to derive how the mechanisms will move in dependence of actuated variables / positions and relations between external forces / torques acting on the flexure and flexural displacements in particular reference systems.

A single elastic part.

Let us describe, at first, flexural characteristics of a general elastic segment separated from a complex flexure as depicted in Fig.3. Mechanical interactions with other / neighboring part of the structure are replaced by internal load and displacements related to references defined to cross-sections in places of interruptions. For simplicity we suppose linear stress – strains dependence i.e. between internal / external forces and deflections. Then, the forces and deflections in the same reference system are related

$$\boldsymbol{d}_B = \boldsymbol{C}_B.\boldsymbol{F}_B = \boldsymbol{S}_B^{-1}.\boldsymbol{F}_B \qquad (1)$$

where:

- $\boldsymbol{d}_B$ is, in general, the six component bi-vector of deflection that consists of three components of translation and three components of rotation column vectors; $\boldsymbol{d}^T = [e_x \quad e_y \quad e_z \quad \varphi \quad \vartheta \quad \psi]$

- $\boldsymbol{F}_B$ is the 6x1 bi-vector of the load that consists of force and moment column vectors; $\boldsymbol{F}^T = [\boldsymbol{f}^T \quad \boldsymbol{m}^T]$

- $\boldsymbol{C}_B$ and $\boldsymbol{S}_B$ are the 6x6 compliance and stiffness matrices of the joint. Components of these matrices are compliance / stiffness coefficients and can be calculated using FEM technique, or, for some specified form of joints / arms applying methods of theory of elasticity especially second Castigliano's theorem [5].

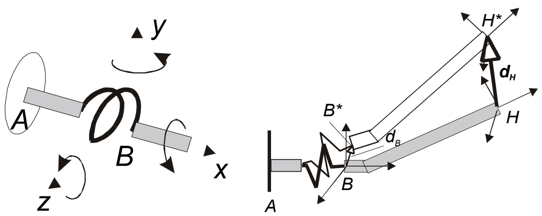


Fig. 3. Deflection of the elastic joint

Express now flexural characteristics of the joint and calculate deflections in *H(x,y,z)* reference system assigned, for instance, to the end of a rigid link body. Define the ${}^B\boldsymbol{T}_H$ the 6x6 transformation matrix between these two reference systems in form

$${}^B\boldsymbol{T}_H = \begin{bmatrix} \boldsymbol{R}_{BH} & \boldsymbol{R}_{BH}.\boldsymbol{P}_{BH} \\ \boldsymbol{0} & \boldsymbol{R}_{BH} \end{bmatrix} \qquad (2)$$

where $\boldsymbol{R}$ is the (3x3) matrix of rotation of the *B* system to the *H* references and $\boldsymbol{P}_{HB}$ is the (3x3) matrix with components p_x, p_y, p_z that are coordinates of the origin *H* with respect to *B* arranged into matrix form

$$\boldsymbol{P}_{BH} = \begin{bmatrix} 0 & -p_z & p_y \\ p_z & 0 & -p_x \\ -p_y & p_x & 0 \end{bmatrix} \qquad (3)$$

Then, for the deflection vectors we can write

$$\boldsymbol{d}_H = {}^B\boldsymbol{T}_H.\boldsymbol{d}_B \qquad (4)$$

and for the transformation of the load force in

$$\boldsymbol{F}_B = \left({}^B\boldsymbol{T}_H\right)^T.\boldsymbol{F}_H \qquad (5)$$

When substitute (1) (2) and (5) into (4) the force – deflection relation in *H* reference system will be

$$\boldsymbol{d}_H = {}^B\boldsymbol{T}_H.\boldsymbol{C}_B.\left({}^B\boldsymbol{T}_H\right)^T.\boldsymbol{F}_H \qquad (6)$$

The compliance matrix related to the H reference system is then

$$\boldsymbol{C}_H = {}^B\boldsymbol{T}_H.\boldsymbol{C}_B.\left({}^B\boldsymbol{T}_H\right)^T \qquad (7)$$

Let us express now the change of transformations as result of elastic deflections of the segment. We consider a typical joint connection of two links according to fig. 4 with dominant coefficients of compliance that correspond rotation around the *z* – axis.

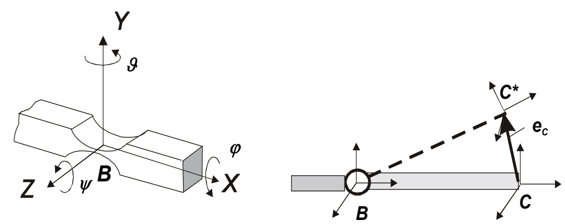


Fig.4. The elastic joint

As was mentioned before there is no ideal elastic joint which exhibits desired deflection in one direction only. Any joint under external load exhibits some cross deflection effects that, in order to satisfy maximal accuracy of mechanisms, as whole, should be considered in precise calculations.
Let us rewrite the deflection vector from (1) or (6) into matrix form according to (2). Then the transformation of the deflected connection of links will be

$${}^B\boldsymbol{T}^*_{C^*} = {}^B\boldsymbol{T}_C.{}^C\boldsymbol{E}_{C^*} \qquad (8)$$

where ${}^C\boldsymbol{E}_{C^*}$ is the (6x6) matrix that represents transformation due to deflections of joint / arm in form

$${}^C E^*_{C^*} = \begin{bmatrix} R_{def} & R_{def}.E_{def} \\ \boldsymbol{0} & R_{def} \end{bmatrix} \qquad (9)$$

in which $\boldsymbol{R}_{def}$ is the (3x3) matrix of rotations with dominant meaning of components that include functions of the desired rotation ψ (see Fig. 4); $c_{(.)}=\cos_{(.)}$, $s_{(.)} = \sin_{(.)}$

$$\boldsymbol{R}_{def} = \boldsymbol{R}_z(\varphi)\,\boldsymbol{R}_y(\vartheta)\boldsymbol{R}_x(\psi) =$$

$$= \begin{bmatrix} c_\varphi c_\vartheta & c_\varphi s_\vartheta s_\psi - s_\varphi c_\psi & c_\varphi s_\vartheta c_\psi + s_\varphi s_\psi \\ s_\varphi c_\vartheta & s_\varphi s_\vartheta s_\psi + c_\varphi c_\psi & s_\varphi s_\vartheta c_\psi + c_\varphi s_\psi \\ -s_\vartheta & c_\vartheta s_\psi & c_\vartheta c_\psi \end{bmatrix} \qquad (10)$$

and

$$E_{def} = \begin{bmatrix} 0 & -e_z & e_y \\ e_z & 0 & -e_x \\ -e_y & e_x & 0 \end{bmatrix} \qquad (11)$$

The other components in (10) (11) represent cross-deflection effects and could be compensated in precise calculations. Naturally, second order terms in (10) can be neglected.

The elastic structure

Let us describe now characteristics of a kinematic mechanisms that consists of rigid parts mutually interconnected by elastic segments. From the structural point of view segments can be arranged in serial, parallel or combined mutual positions. There are:

a) *Segments are in serial configuration* (Fig. 5)

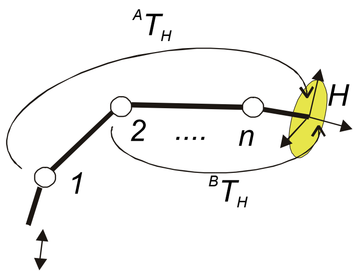


Fig. 5. Serial configuration of elastic segments

The goal is to describe how the chain of segments will deflect under acting of external force (or displacement) expressed in H references.
As follows from our previous analysis the contribution of a particular segment will be

$$\boldsymbol{d}_{H(i)} = {}^{i}\boldsymbol{T}_{H}^{*}.\boldsymbol{d}_{i} \qquad (12)$$

The end deflection is given by superposition of particular deflections in all segments. Then, one can write

$$\boldsymbol{d}_{H} = \sum_{(i)} {}^{i}\boldsymbol{T}_{H}^{*}.\boldsymbol{d}_{i}\ ; \quad i = 1,2,\ldots n \qquad (13)$$

and

$$\boldsymbol{F}_{i} = \left({}^{i}\boldsymbol{T}_{H}^{*}\right)^{T}.\boldsymbol{F}_{H} \qquad (14)$$

$$\boldsymbol{d}_{i} = \boldsymbol{C}_{i} \cdot \boldsymbol{F}_{i} \qquad (15)$$

where obviously

$${}^{i}\boldsymbol{T}_{H}^{*} = {}^{i}\boldsymbol{T}_{i+1}^{*} \cdot {}^{i+1}\boldsymbol{T}_{i+2}^{*} \cdot \ldots \cdot {}^{n}\boldsymbol{T}_{H}^{*} \qquad (16)$$

When substitute (14) and (15) into (13) one can write force – deflection relationship in form

$$\boldsymbol{d}_{H} = \sum_{n} \left[{}^{i}\boldsymbol{T}_{H}^{*} \cdot \boldsymbol{C}_{i} \cdot \left({}^{i}\boldsymbol{T}_{H}^{*}\right)^{T} \right] \cdot \boldsymbol{F}_{H} \qquad (17)$$

from where the compliance of the arm which consists of serial segments is

$$C_{H} = \sum_{n} {}^{i}\boldsymbol{T}_{H}^{*} \cdot \boldsymbol{C}_{i} \cdot \left({}^{i}\boldsymbol{T}_{H}^{*}\right)^{T} \qquad (18)$$

b) *Segments are arranged by parallel way*

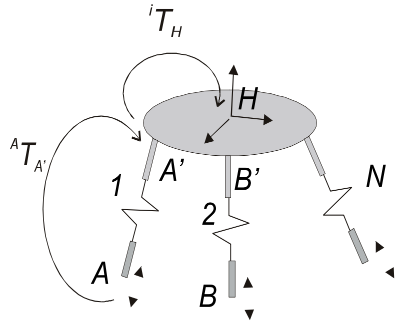


Fig. 6. Parallel arrangement of elastic segments

Express now flexural characteristics for parallel configuration of segments. For such structures one can write

$$\boldsymbol{F}_{H} = \sum_{(i)} \left({}^{i}\boldsymbol{T}_{H}\right)^{T} \cdot \boldsymbol{F}_{i} \qquad (19)$$

$$\boldsymbol{d}_{i} = \left({}^{i}\boldsymbol{T}_{H}\right) \cdot \boldsymbol{d}_{H} \qquad (20)$$

Because of $\boldsymbol{F}_{i} = \boldsymbol{S}_{i} \cdot \boldsymbol{d}_{i}$, $\boldsymbol{S}_{i}$ is the stiffness matrix of a segment, similarly as in previous case we get

$$\boldsymbol{F}_{H} = \sum_{(i)} \left({}^{i}\boldsymbol{T}_{H}{}^{T} \cdot \boldsymbol{S}_{i} \cdot {}^{i}\boldsymbol{T}_{H} \right) \cdot \boldsymbol{d}_{i} \qquad (21)$$

c) *Combined configuration of elastic segments: serial and parallel.* (see Fig.2a,b).

Real kinematic flexures are usually created as structures of interconnected elastic segments and rigid parts in serial and parallel configuration. Then, calculation of flexural characteristics combines both above procedures.

4. Example of the x-y positioning table

The analysis of the 2 d.o.f. planar mechanism according to Figs.7 and 8 is made as illustrative example.

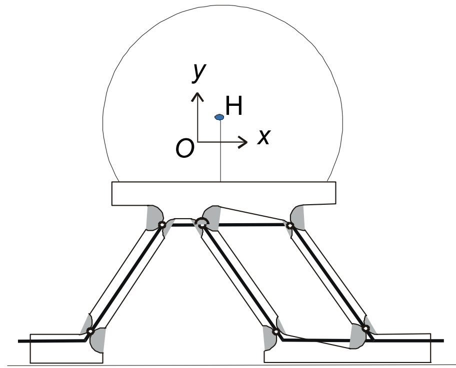


Fig.7. The elastic mechanisms

The flexural kinematics exhibits x-y positioning capability of the table as result of two actuated linear displacements x_A, x_B of points A,B with respect to their original positions A_o and B_o and constructional distance A_oB_o.

.

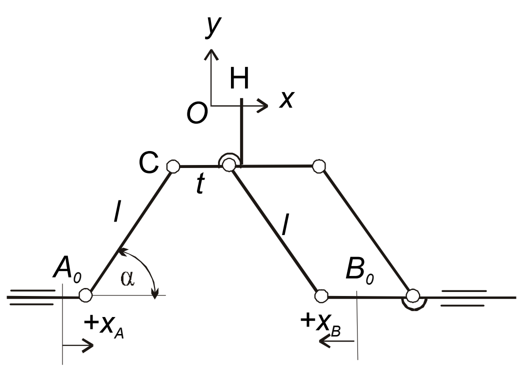


Fig.8. Kinematic structure

Let us denote the stable coordinate system O(x,y) in which movements of the table reference point H are evaluated. Then, displacements of the table will be

$$x_H = x_A - x_B + l.(\cos\alpha - \cos\alpha_0) \qquad (22)$$
$$y_H = l.(\sin\alpha - \sin\alpha_0)$$

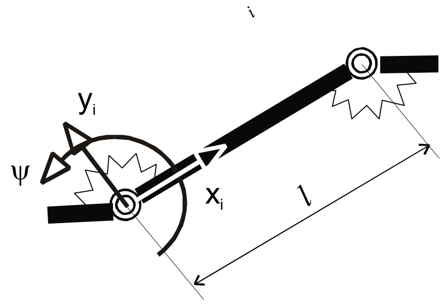
where

$$\cos\alpha_0 = \frac{\overline{A_0B_0} - t}{2l} \; ; \; \cos\alpha = \frac{\overline{A_0B_0} - t - x_A - x_B}{2l}$$

$$\sin\alpha_0 = \sqrt{1-\cos^2\alpha_0} \; ; \; \sin\alpha = \sqrt{1-\cos\alpha}$$

The task of flexural analysis, as described before, is to calculate the directional stiffness, or the compliance, of this mechanism with respect to forces F_A / F_B acting on the flexure in that correspond directions x_A / x_B. This driving force should be exerted to deflect the spring-like mechanism and resulting movements of the table. It is obvious that F_A = - F_B if there is no loading force on the table.

Let us divide the mechanism and separate one of arms with two flexible joints according to Fig. 9. Because of planar mechanism all loads and deflections will be considered in the plane only. Denote



5. Conclusion

The paper presents a procedure for calculation flexural characteristics of kinematics where classic revolute joint connections of links are replaced by elastic joints. A systematic approach to the analysis of flexures is given. By the presented way it is possible to built the model of the whole flexure. The model can include calculations / simulation of important characteristics, as follows:

- Driving forces and displacements that depend on actual position of the mechanisms / internal energy of elasticity.
- Optimization of the form and parameters of flexural segments as well as geometry.
- Working volume and potential functions.
- System dynamics, etc.

These problems are tasks for elaboration in the future.

References

[1] http://dmtwww.epfl.ch/isr/hpr, http://roboticseecs.berkeley.edu, http:/www.iwf.ing.tu-bs.de, http://micron.me,dal.ca,

[2] Carbone, M.- Ceccarelli, M.- Kerle, H.: Designing Mechanisms for Two-finger Microgrippers. ‚Proc. RAAD‘2001, Workshop on Robotics in Alpe-Adria-Danube Region, May, 16-19, Vienna‘

[3] Hickey, R.- Hubbard, T.- Kujath, M.R.: Development of micromanipulators using MEMS technology. „Proc. RAAD‘2001, Int. Workshop, Robotics in Alpe-Adria-Danube Region," May, 16-19, Vienna

[4] Campos M.F.M.: Robotic exploration of material and kinematic properties of objects. PhD Disertation, MS-CIS-92-38, Grasp Lab 316, Pensylvania State University 1992

[5] Havlík, Š.: A new elastic structure for a compliant wrist. **ROBOTICA** 1983, Vol.1, pp. 95-102, GB

[6] Kallenbach, E. et al.: New polarized electromagnetic actuators as integrated mechatronic components – design and application. „Mechatronics," Vol. 9 (1999) No. 7, pp. 769 – 784

[7] Jeckel M. and Havlik, S.: An approach to design of robotic effectors for grasping soft and fragile parts. Proc. RAAD'2002, Workshop, Robotics in Alpe-Adria-Danube Region,