

Passive Compliant Mechanisms for Robotic (Micro)Devices

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Abstract—*The paper deals with problems of designing passive compliant mechanisms for some robotic devices as: grippers - accommodators, or multi-component force/displacement sensors. The evaluation and comparing flexural characteristics of compliant mechanical segments: joints, arms or whole structures are analyzed. The design study of a compliant grasping mechanism that create RCC grasp of a part is discussed and procedure for synthesis the geometry of elastic finger structure is presented*

Keywords: accommodation, compliant grasp, elastic structure, stiffness

I. Introduction

Compliant mechanical structures and mechanisms represent a broad class of mechanical systems where displacements of end parts are resulted in elastic deflection of their flexural joints, or link segments. The meaning and application of compliant mechanisms is getting more and more important especially in constructions of precise positioning mechanisms or small and micro-scale elastic structures, where classic constructions from discrete parts are hardly realizable. The single solution lies in design of compact compliant mechanical structures and using appropriate advanced manufacturing technology. Compact designs and MEMS technologies enable to miniaturize dimensions and to manufacture such structures in small or, micro scale dimensions.

In designing any mechanisms, at the beginning, there is always a first intuitive proposal. As to compliant mechanisms the first proposal of the structure usually goes out from the similarity with some known rigid-body mechanisms. Naturally, designing more complex compliant structures that include elastic and/or relatively rigid elements suppose using techniques for force and compliance analysis, modeling and simulation of flexible structures as well. The final design is then always a choice of geometry and parameters that satisfy some optimal / compromise solution. It should be said that designing a compliant mechanisms much more attention and effort should be devoted to this design phase then in cases of classic mechanisms.

Robotic devices, we have in mind, basically consist of elastically compliant mechanisms that should satisfy specific characteristics and satisfy criteria given by device and its application. Such devices are multi-component force and torque sensors, precise or micro-positioning robotic tools and effectors [1,2,3,4,5,6,7,8,9]. For designing compliant structures / mechanisms in both classes of these devices there are similar problems that result in using the same theory and common approaches to analysis, modeling and performance evaluation can be applied in the design process.

II. Compliant mechanisms in robotic devices

There are two groups of compliant mechanisms according to the input energy exerted for performing output motions: active and passive. Active devices work similarly as classic robot arms (mainly parallel), where particular joints are actuated. On the other hand; passive devices deflect under external forces and the desired output motion is given by their deformable structure. Such devices are frequently used as compliant grippers / fingers or accommodators for compensation of errors due to misalignments in positioning. The broad class of passive compliant devices represent multi-component force or displacement sensors. The common feature of these devices is that they have a limited range of motions given by form and material characteristics of their elastic parts / structures.

Designing elastically compliant structures of such kind of compliant mechanisms there are, in principle, two ways:

- The kinematic approach where the structure corresponds to classic mechanisms; where only revolute joints are replaced by elastic hinges.
- The distributed flexibility approach, i.e. the final structure consists of both compliant arms and/or joints. The end motion is realized by deflections of the whole structure.

The difference between these two approaches can be seen on examples of solving two RCC (Remote Center Compliance) mechanisms in Fig.1 [1, 2]. Naturally, both approaches can be combined in one complex structure.

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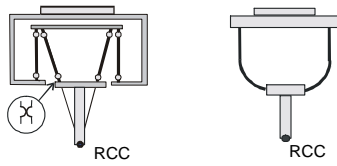


Fig.1. Two concepts of the passive RCC mechanisms

A. Positioning devices

Talking generally about compliant positioning devices mechanisms that perform desired motion of the end part under acting some external forces are considered. In case of active mechanisms the position and motion of the end part is controlled via measured input displacements of driven members. The accuracy of the end position corresponds to positional accuracy of active members; as input variables. On the other hand; the passive compliant mechanisms deflect under acting of external forces / moments and motion of the end part solely depends on flexural characteristics of their deformable structure. This fact naturally results in demand of much more careful design of the deformable structure. This practically means that methods of flexural analysis, synthesis and optimization procedures should be used in design. The design task then consists of two steps:

- Design of mechanical structure able, in principle, to perform desired motions.
- Applying optimization procedures in order to find parameters that satisfy several criteria that compliant mechanisms should satisfy. There are, for instance, as follows:
 - Accessibility to all points of the operation space, especially points on borders when forces, or, maximal strains in joints are limited.
 - Minimal “dead” energy of elasticity accumulated in elastic structure.
 - Desired ratio of displacement or force transmissions.
 - Other task related criteria.

B. Multicomponent force - displacement sensing devices

Each sensor consists of three main functional parts: mechanics, transducers and data/signal processing circuitry [3].

Designing any sensor there are two main decisive steps that must be solved: choosing the sensing principle with adequate method of processing signals and an appropriate mechanical structure. It is obvious that both problems are closely related. As far the sensing principle and transducers were chosen the role of sensor mechanics is to produce measurable strains / displacements. The form and geometry of the elastic body and configuration of transducers is the main task for the design of sensor mechanics. It should be said that design of mechanical structure directly corresponds to correct function and quality of the sensor in static and dynamic mode of use. For this reason it is very important to pay attention to

analysis, modeling and design of compliant sensor structures.

III. Analysis of compliant structures

A. Elastic segments

Any compliant mechanism consists of elastic and rigid segments mutually interconnected in one compact structure [10,11,12]. The fundamental parts that are elastic segments with characteristics that closely relate to final accuracy of the compliant structure.

Describe flexural characteristics of an elastic segment / joint separated from a compact structure [13,14].

Mechanical interactions of such an elastic segment with other / neighboring part of the structure are replaced by internal load and displacements related to references defined to cross-sections in places of interruptions. For simplicity we suppose linear stress – strain dependence i.e. linear relations between internal / external forces and deflections. The assumption of linearity is valid for majority of classic elastic materials; as spring steel, glass, poly-sillicium, etc., frequently used for fabrication of small size mechanisms. Then, the forces and deflections in the same reference system are related

$$\mathbf{d} = \mathbf{C} \cdot \mathbf{L} = \mathbf{S}^{-1} \cdot \mathbf{L} \quad (1)$$

where

$-\mathbf{d}^T = [d_x \ d_y \ d_z \ \delta_x \ \delta_y \ \delta_z]$ is, in most general case, the six component vector of deflection that consists of three components of translation and three components of rotations,

$-\mathbf{L}^T = [f_x \ f_y \ f_z \ m_x \ m_y \ m_z]$ is the six component vector of the load that consists of force and moment components,

$-\mathbf{C}$ and \mathbf{S} are the (6x6) compliance and stiffness matrices respectively.

One of the crucial problems in designing flexural structures is how to compare various deformable segments as to their flexural characteristics.

For such linear systems it is possible to analyze and compare them using method of singular value decomposition (SVD). Remark: SVD is the general method for examination characteristics of the linear transformations.

The SVD of the compliance matrix \mathbf{C} from (1) is expressed by transformation

$$\mathbf{C} = \mathbf{G} \cdot \mathbf{\Phi} \cdot \mathbf{H}^T \quad (2)$$

where \mathbf{G} , \mathbf{H}^T are orthogonal matrices and $\mathbf{\Phi} = \text{diag}(\varphi_1, \varphi_2, \dots, \varphi_6)$ are singular values of the compliance matrix.

The geometric interpretation of such analysis is as follows:

We define the unit sphere in the force space. Using transformation (1) the unit sphere is mapped into the deflection space as a generalized ellipsoid. The lengths of

its main axes are singular values with orientation given by columns of the G matrix (See Fig.2).

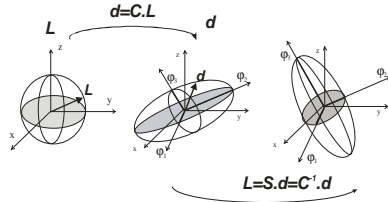


Fig. 2. The compliance and stiffness ellipsoids

Then, comparing elastic segments, two bodies will have the same flexural features, if they exhibit the same compliance / stiffness ellipsoids, as to length of its axes and their orientation, as well. Various forms of elastic parts can exhibit a given selective compliance in particular directions.

Mathematically; the compliance matrix, except the dominant coefficient, includes other unwanted compliance elements and the compliance ellipsoid exhibits some finite not negligible volume. These effects naturally deteriorate the accuracy of joint motion and result in worse positional accuracy of the final mechanism. Comparing to “ideal revolute or prismatic connections” any real elastic joint always exhibits some lumped - cross flexural effects, i.e. beside the desired motion it deflects in other directions too. Then, according to axes of compliance ellipsoids (as to lengths and orientations) it is possible to evaluate the “kinematic quality” of particular flexural parts / elastic joints. As obvious, the volume of the inverted – stiffness ellipsoid corresponds to the energy of elasticity i.e. (+/-) work for deflection that should be exerted for desired motion.

It should be said that in order to satisfy maximal accuracy of the mechanisms these effects of unwanted deflections should be considered in precise calculations. Then, they can be eliminated, minimized, or otherwise compensated. This is the optimization task in designing forms, geometry and parameters of elastic bodies.

The following TABLE 1 shows comparison of the kinematic quality of joints / elastic hinges frequently used in compliant structures.

	Compliance axes $\zeta = \psi \gg \xi$
	Compliance axes $\zeta \gg \psi, \xi$
	Compliance axes $\zeta > \xi \gg \psi$

TABLE 1. Compliance characteristics of some elastic joints

B. Compliant Structures

Let us describe now characteristics of a kinematic mechanism that consists of rigid parts mutually interconnected by elastic segments. From the structural point of view segments can be arranged in serial, parallel or combined mutual positions. The goal is to describe how the chain of segments will deflect under acting of external load, both expressed in end frame [13].

In serially arranged elastic segments the end deflection is given by superposition of particular deflections of all segments. The end compliance matrix C_H related to the end H reference system the force – deflection characteristics is then

$$C_H = \sum_n {}^i T_H^* \cdot C_i \cdot ({}^i T_H^*)^T \tag{3}$$

where ${}^i T_H^*$ are deformed transformation matrices between finger contact and end reference systems.

For segments arranged in parallel configuration the end stiffness is calculated

$$S_H = \sum_{(n)} ({}^i T_H^T \cdot S_i \cdot {}^i T_H) \tag{4}$$

Because of compliant mechanisms are usually created as compact flexural structures of mutually interconnected elastic segments and rigid parts in serial or/and in parallel configuration, calculation of flexural characteristics combines both above procedures.

<p>Revolute joint</p>	$\zeta \rightarrow \infty ;$ $\psi, \xi = 0$
	Compliance axes $\zeta \gg \psi, \xi$

IV. The gripper with compliant fingers for insertion tasks. The design study.

The task is to design a robotic (micro) gripper that exhibit the error self-compensation capability in its own compliant structure. Many devices for this purpose have been yet designed are well known as RCC passive compliant wrists. Such mechanical devices were especially designed for “peg in hole” assembly many years ago [1,2]. This operation was analyzed in details and procedures of insertion were elaborated and experimentally studied, as well [15,16]. Unfortunately the concept of a compliant wrist can not be used for fast speed manipulations with mini or micro parts, as for instance: insertions miniature or fragile pegs with diameters less than 1mm, screwing small screws, etc. This requirement gives motivation for further study of devices able to work similarly with small and micro parts [17,18,19]. The single solution lies in designing some grasping mechanisms where RCC features are inherent in fingers of a gripper. Thus, the mass that loads the elastic structure is minimized what results in higher accuracy / higher frequency of the positioning system.

Designing this device the concept of elastic fingers has been adopted. This concept enables to minimize mass and dimensions of the gripper, as well.

A. Compliant grasp

Consider a part (peg) has to be grasped, transported and mated together with another part (hole) by a closely fitting operation. In principle, the grasping mechanism consists of several elastic fingers, as depicted in Fig. 3.

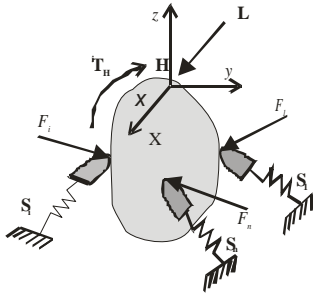


Fig. 3. Compliant grasp

The stability of such grasp during mating operation is assured if following force condition is satisfied

$$\vec{L} \leq \sum_n {}^i T_H \cdot \vec{F}_i \quad (5)$$

where L is the six component vector of external forces and moments and F_i are six / component vectors of contact forces and moments between fingers and object surface and T is the force transformation matrix between H reference system and references in contact of fingers. As the fingers are elastically compliant the contact force is function of their stiffness S and deflection d vector

$$F_i = S_i \cdot d_i \quad (6)$$

B. Mathematical formulation of the design problem

The compliant fingers should compensate small lateral and angular misalignments during mating. Principal requirement is that deflection of the compliant structure should not deteriorate relative position of parts to be mated together. The compliance center of a flexure is the point where the acting force results in pure translation and the moment results in pure rotation of the end part.

Mathematically this “RCC” feature can be formulated as follows:

Let L is the vector of the external force and moment; d is the six component vector of elastic deflections (3 components of translation, 3 components of rotation), both in H reference system assigned to the contact point in compliance center. Then; the RCC feature of such structure is expressed by relation

$$d = C_{RCC} \cdot L \quad (7)$$

where C_{RCC} is the compliance matrix that include only diagonal non-zero elements, as compliance coefficients.

C. Design of compliant grasp structure

The designed compliant grasp structure, as depicted in Fig. 4 consists of three elastic fingers centrally actuated. The concept of curved elastic rods that create similar compact flexure was originally designed for a compliant RCC robot wrist [2]. The fingers in form of thin elastic rods are made up from of the straight section of length l and the circular arc defined by radius ρ and by angle ϕ . The finger grasp-structure and its main geometrical parameters are. Cross section of fingers can be circular (wires with diameter d_1), or rectangular (strips with $b \times h$ cross section) and corresponds to desired performance and manufacturing technology of gripper. For simplicity we suppose wires in the first design. The fingers are regularly arranged on the circle with diameter d . The last geometrical parameter is the distance of the remote center a (or a^*). These six parameters: l, d, ρ, ϕ, d_1, a determine the compliant structure and together with material characteristics (Young’s elasticity module E , and Poisson’s ratio μ) its compliance characteristics. It is obvious, that such compliant grasp structure enables small flexural movements of the part (peg) in six D.o.F.s.

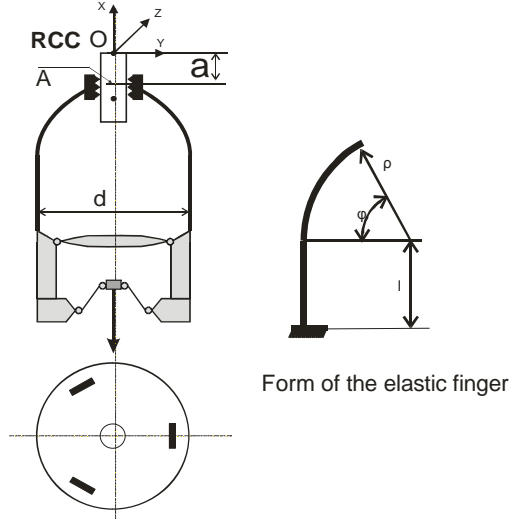


Fig. 4. Compliant RCC grasp structure

As supposed, functioning of such RCC device is sketched in Fig. 5.

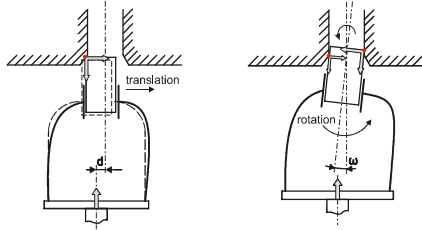


Fig. 5. Two phases of "a peg into a hole" insertion

D. Analysis and synthesis of the structure

In order to describe the compliance characteristics of this closed grasp structure in form of (4) the compliance / stiffness matrices of the single finger should be derived.

As the first step let us take out one finger and contact reactions with object replace by internal force and moment vectors in local coordinates related to the end of finger.

In the most general case the compliance matrix C_i (6x6), or, stiffness S_i (6x6) of the single finger is function of its geometrical parameters l, ρ, ϕ, d_I .

Several methods of the theory of elasticity that can be applied for calculation of particular compliance coefficients. There are for instance:

- Expression of the stress energy and applying Castigliano theorems [2].
- Using FEM and available SW tools.

The product of calculation is the finger compliance or stiffness matrix related to local references in O_i . Then, the compliance and stiffness (4) of the whole grasp structure was calculated for a chosen set of six

geometrical parameters [2]. The compliance matrix (3) of the grasp structure related to the reference system which is parallel to $O(x,y,z)$ will include coefficients

$$C_H = \begin{bmatrix} c_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{22} & 0 & 0 & 0 & c_{26} \\ 0 & 0 & c_{33} & 0 & c_{35} & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & c_{53} & 0 & c_{55} & 0 \\ 0 & c_{62} & 0 & 0 & 0 & c_{66} \end{bmatrix} \quad (8)$$

As follows from the compliance analysis due to rotational symmetry of the finger structure we have

$$c_{22} = c_{33}, \quad c_{55} = c_{66} \quad \text{and} \quad c_{26} = c_{35}, \quad c_{62} = c_{53} \quad (9)$$

The second step of the design procedure is synthesis the geometry with the goal to find such a suitable combination of geometrical parameters l, d, ρ, ϕ, d_I, a that satisfy RCC characteristics and desired functional requirements. Thus the desired diagonal form of the compliance matrix it is the necessary to satisfy

$$c_{62} = 0 \quad \text{and} \quad c_{26} = 0 \quad (10)$$

The adequate condition we get after a simple force-kinematic consideration (see Fig. 4).

We have the compliance matrix C_A calculated to the parallel reference system at the point A. As denoted the distance a of the RCC point is defined between these two reference systems.

Suppose that only external radial force f_y is acting on the RCC tip. The effect of this action should be purely translational deflection, i.e. there is no rotation around the z axis in our parallel system in point A.

Let us express this condition mathematically and denote by left upper indexes A / O assigned to load components and compliance coefficients to which coordinate system they belong. According to the above condition pure translation becomes if the following equation written for A system is satisfied

$${}^A\delta_z = {}^A c_{62} \cdot {}^O f_y + {}^A c_{66} \cdot ({}^O f_y \cdot a + {}^A m_z) = 0 \quad (11)$$

Because of $m_z=0$, the condition which is equivalent to (10) will be

$${}^A c_{62} + a \cdot {}^A c_{66} = 0 \quad (12)$$

When express small deflection / translation due to the action of force component ${}^O f_y$, we get

$${}^A d_y = ({}^A c_{22} + a \cdot {}^A c_{62}) \cdot {}^O f_y \quad (13)$$

Considering (9) and because of ${}^A c_{66} = {}^O c_{66}$, we substitute condition (12) into (13). Then translational deflection of such RCC structure will be

$${}^o d_y = {}^A d_y = ({}^A c_{22} - a^2 \cdot {}^A c_{66}) \cdot f_y \quad (14)$$

From this equation the dependence between compliance coefficients in both parallel reference systems is

$${}^o c_{22} = {}^A c_{22} - a^2 \cdot {}^A c_{66} \quad (15)$$

The final design of the structure then includes the procedure of searching within the space of constructional values of geometrical parameters until diagonality condition (12) and a satisfactory function is achieved. By defining suitable error functions some optimization techniques can be used as algorithms. An example of synthesis procedure shows scheme in Fig. 6 and result characteristics of a RCC grasp flexure is shown below.

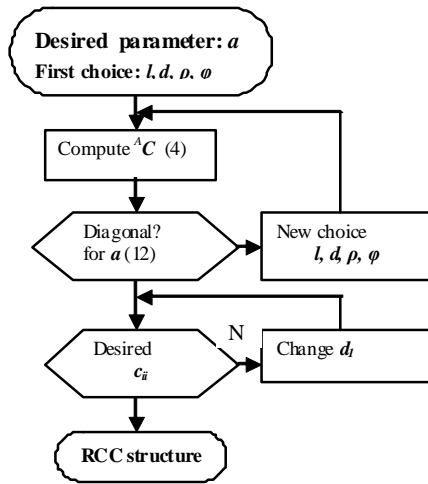


Fig.6. Computational scheme of parameter synthesis

The structure is calculated for the model (10:1 scale).
Given values:

- The distance of the RCC:..... $a = 100$ mm,
- The radial compliance: ${}^o c_{22} = 0,04$ mm/N,
- Number of fingers: 3

Results of synthesis / optimization procedure, as described above, are computed parameters that specify geometry of the proposed elastic grasp structure.

The geometry and dimension of the finger structure:

$$l = 10 \text{ mm}, \rho = 30 \text{ mm}, \varphi = 40^\circ, \text{ and} \\ d = 80 \text{ mm}, d_1 = 1,0 \text{ mm}.$$

Flexural characteristics of this grasp structure give compliance matrices related to reference systems in O (RCC) and A points.

The compliance matrix in the RCC is:

$${}^o C_{RCC} = \begin{bmatrix} 1,710 & 0,0 & 0,0 & 0,0 & 0,0 & 0,0 \\ 0,0 & 39,843 & 0,0 & 0,0 & 0,0 & 0,0 \\ 0,0 & 0,0 & 39,843 & 0,0 & 0,0 & 0,0 \\ 0,0 & 0,0 & 0,0 & 0,147 & 0,0 & 0,0 \\ 0,0 & 0,0 & 0,0 & 0,0 & 0,0297 & 0,0 \\ 0,0 & 0,0 & 0,0 & 0,0 & 0,0 & 0,0297 \end{bmatrix} \cdot 10^{-3}$$

and the compliance matrix of the finger structure related to the parallel reference system in A point:

$${}^A C = \begin{bmatrix} 1,710 & 0,0 & 0,0 & 0,0 & 0,0 & 0,0 \\ 0,0 & 45,163 & 0,0 & 0,0 & 0,0 & 0,0525 \\ 0,0 & 0,0 & 45,163 & 0,0 & 0,0525 & 0,0 \\ 0,0 & 0,0 & 0,0 & 0,147 & 0,0 & 0,0 \\ 0,0 & 0,0 & 3,010 & 0,0 & 0,0297 & 0,0 \\ 0,0 & 3,010 & 0,0 & 0,0 & 0,0 & 0,0297 \end{bmatrix} \cdot 10^{-3}$$

Particular coefficients correspond to six-component vectors (1): the deflection vector (3 translations, 3 rotations) and load vector (force, moment) with units

$$\begin{bmatrix} mm/N & mm/N.mm \\ deg/N & deg/N.mm \end{bmatrix}.$$

V. Conclusions

Compliant (active and passive) mechanisms are widely applied in many robotic devices as for instance: precise micro-positioning devices, grippers and tools for manipulation with small, soft or fragile objects, accommodation devices, etc. These mechanisms are usually created as compact elastic structures frequently made from one piece of elastic material. Meaning and application of compliant mechanisms is getting more important especially together with the development of micro-technologies and automation. The broad groups of such devices are multi-component force and displacement sensors with sensing principles based on measurement deflections (strains, mechanical displacements) on elastically compliant structures.

The important feature of devices based on passive principle is that their performance strictly depends on flexural characteristics of compliant mechanics. For this reason very careful design, including analysis of elastic structures and modeling techniques, is strongly recommended. Robotic devices built on passively compliant mechanisms represent a reliable and relatively low cost group of equipment for automation.

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